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TRIANGULAR NUMBERS AND ILLUSTRATIONS

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N. Kasthuri	Assistant Professor, Department of Mathematics, VMKV Arts & Science College, Salem (India)

Abstract

This paper focus on Triangular Numbers and illustrations. The triangular numbers, which are numbers associated with certain arrays of dots, were known to the ancient Greeks and viewed by them with reverence. Though possessing a simple definition, they are astonishingly rich in properties of various kinds, ranging from simple relationships between them and the square numbers to very complex relationships involving partitions, modular forms, etc. — topics which belong to advanced mathematics. They also possess many pretty combinatorial properties. In this expository article we survey a few of these properties.

1. INTRODUCTION TO TRIANGULAR NUMBERS

The numbers in a sequence are referred to as terms. The terms of a triangular sequence are related to the number of dots needed to create a triangle.

The forming a triangle with three dots; one on top and two on bottom. The next row would have three dots, making a total of six dots. The next row in the triangle would have four dots, making a total of 10 dots. The following row would have five dots, for a total of 15 dots. Therefore, a triangular sequence begins: “1, 3, 6, 10, 15…”

Triangular numbers, as shown in the image here, are a pattern of numbers that form equilateral triangles. Each subsequent number in the sequence adds a new row of dots to the triangle. It is important to note that in this case, n equals the term in the sequence. So, n equals 1 is the first term, n equals 5 is the fifth term, n equals 256 is the 256th term. Believe it or not, we can actually use this n to figure out how many dots are in its corresponding triangle (i.e. its triangular number).

2. REPRESENTATION

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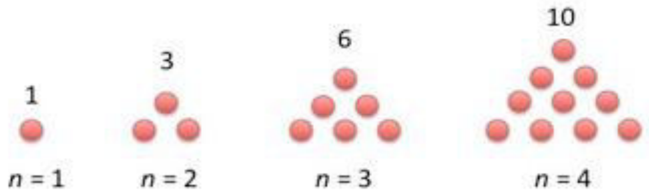

Here T_n denotes the number of dots is the nth such array. Then

$T_1 = 1$, $T_2 = 3$, $T_3 = 6$, $T_4 = 10$, $T_5 = 15$,……

A triangular shape can be constructed using dots only if the number of dots is a triangular number. It should be fairly clear that,

$$T_n = 0+1+2+3+\ldots\ldots+(n-1)+n$$

For each new row of dots contains one more dots than the previous row. We obtain more members of the sequence T. Here is the sequence

T = { 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55,……}

<u>To find the formula for T_n the nth triangular number</u>

We apply the method of differences to T and obtain the following array:

0	1	3	6	10	15	21
1	2	3	4	5	6	
1	1	1	1	1		

The sequence of 2nd difference consists only of 1's, so we ma suppose that the leading term in the generating formula for T is some constant for n^2 so the leading term in the generating formula ought to be $n^2/2$.

We compute the sequence $A = T - n^2/2$ and we do a term by term subtraction of { 0, ½, 2, 9/2, 8, 25/2, 18, 49/2,….. }

Form T = { 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55,……}

And obtain the sequence $A = T - n^2/2$

We get A = {0, 1/2, 2 , 3/2, 2, 5/2, 3, 7/2, ………… }

This sequences can be written in a nicer from by expressing all the number as fraction with a denominator of 2:

A = {0, 1/2, 2 , 3/2, 2, 5/2, 3, 7/2, ………… }

And it appears from that A is just {n/2}

The sequence initially given must have been { $n^2/2 + n/2$}

(i.e) $\frac{n(n+1)}{2}$

The conclusion is thus, $T_n = 0+1+2+3+\ldots+n$

$$T_n = \frac{n(n+1)}{2}$$

(i.e) the nth triangular number is $\frac{n(n+1)}{2}$

Example:1

For n = 10 the formula states that

T_{10} = 0+1+2+3+…….+ 10

T_{10} = 55

For n = 15

T_{15} = 0+1+2+3+…….+ 15

T_{15} = 120

3. RELATIONSHIP BETWEEN THE T – NUMBER AND THE SQUARES

Relationship between the T – Number and the Squares is obtained by multiplying the T – numbers by 8 and adding 1 to the result.

For T = { 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55,……}

We obtain the numbers {9, 25, 49, 81, 121, 169, 225, 289, 361, ……….}

And these are just the odd squares

(i.e) For T_1 = 1

we have 8 T_n + 1 = 8 x T_1 + 1

= 8 x 1 + 1

= 8 T_1 + 1 = 9

Similarly for T_4 = 10

we have 8 T_4 + 1 = 8 x T_4 + 1

= 8 x 10 + 1

= 8 T_4 + 1 = 81

Hence we concluded that the relation between the T – numbers and the squares is that

8 T_4 + 1.

Statement :

If x is an odd square then $\frac{X-1}{8}$ is a T - number

Proof:

If x is an odd square then $X = (2n + 1)^2$

(i.e) $X = (2n + 1)^2$

$X = 4n^2 + 4n + 1$, for some integer 'n'

But now we get

$$\frac{X-1}{8} = \frac{4n^2+4n+1-1}{8} = \frac{4n^2+4n}{8} = \frac{n^2+n}{2}$$

$$= \frac{n(n+1)}{2}$$

$$\frac{X-1}{8} = T_n$$

Hence if 'X' is an odd square then $\frac{X-1}{8}$ is a T - number

Example :2

From the odd square 169 we get the T - numbers as follows

(i.e) X = 169 we get the T – number

$\frac{X-1}{8} = \frac{169-1}{8} = \frac{168}{8} = 21$

$\frac{X-1}{8} = T_6$

Example :3

From the odd square 841 we get the T - numbers as follows

(i.e) X = 841 we get the T – number

$\frac{X-1}{8} = \frac{841-1}{8} = \frac{840}{8} = 105$

$\frac{X-1}{8} = T_{14}$

Problem :1

Compute the cumulative sums of the reciprocals of the T – numbers. Denote by a_n the some representative value of the sum

$\frac{1}{T1} + \frac{1}{T2} + \ldots\ldots\ldots\ldots + \frac{1}{Tn}$

We have $a_1 = \frac{1}{T1}$ $a_2 = \frac{1}{T1} + \frac{1}{T2}$

$a_1 = \frac{1}{1}$ $a_2 = \frac{1}{1} + \frac{1}{3}$

$a_1 = 1$ $a_2 = \frac{4}{3}$

$a_3 = \frac{1}{T1} + \frac{1}{T2} + \frac{1}{T3}$

but $a_2 = \frac{1}{T1} + \frac{1}{T2}$

so $a_3 = a_2 + \frac{1}{T3}$

$= \frac{4}{3} + \frac{1}{6} = \frac{3}{2}$

$a_3 = \frac{3}{2}$

Similarly,

$a_4 = \frac{3}{2} + \frac{1}{10} = \frac{8}{5}$

$a_5 = \frac{8}{5} + \frac{1}{15} = \frac{5}{3}$

$a_6 = \frac{5}{3} + \frac{1}{21} = \frac{12}{3}$ and so on

If we have the numbers a1, a_2, a_3 ……. we get the fractions

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7} \ldots.$

We have,

$\frac{an}{2} = \frac{n}{n+1}$

$\frac{1}{T1} + \frac{1}{T2} + \ldots\ldots. + \frac{1}{Tn} = \frac{2n}{n+1}$

From above equation we written as,

$$\frac{2n}{n+1} = 2 - \frac{2}{n+1}$$

And this quantity gets gradually closer to as 'n' grows and larger, because $\frac{2}{n+1}$ grows smaller and smaller as 'n' grows.

Hence we conclude that as 'n' grows, the sum of the reciprocals of the first 'n' triangular numbers gets arbitrarily close to 2. Hence the sum of the reciprocals of all the triangular numbers equal 2.

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Authored by

N. Kasthuri

Assistant Professor, Department of Mathematics,
VMKV Arts & Science College, Salem (India)

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